From ChatGPT:

Consider the arithmetic series of the first 'n' natural numbers:

1 + 2 + 3 + ... + (n-2) + (n-1) + n

Now, let's use the "pairing" or "averaging" technique to simplify the calculation:

Pairing the terms from both ends of the series, we get the following pairs:

(1 + n) + (2 + (n-1)) + (3 + (n-2)) + ... + ((n-1) + 2) + n

Each pair in the parentheses sums to 'n+1'. Now, if we count the total number of pairs, we find that there are exactly 'n' pairs in the series.

The Pairing Technique:

The pairing technique involves pairing up the terms of the arithmetic series in a way that each pair sums to the same value, which is the sum of the first and last terms. The technique simplifies the calculation of the sum, making it more efficient.

Pairing the Terms:

For an arithmetic series of the first 'n' natural numbers, we can pair up the terms from both ends. The first term is paired with the last term, the second term is paired with the second-to-last term, and so on.

For example, with 'n = 5', we have the series: 1 + 2 + 3 + 4 + 5

Pairing: (1+5) + (2+4) + 3

Sum of Each Pair:

In each pair, the sum of the two terms is equal to (n + 1). For 'n = 5', each pair sums to 6 (5+1).

Number of Pairs:

The total number of pairs in the series is half of the total number of terms. When 'n' is even, we have 'n/2' pairs, and when 'n' is odd, we have '(n-1)/2' pairs.

Calculating the Sum:

To calculate the sum of the series using the pairing technique, we can simply multiply the sum of each pair (n + 1) by the number of pairs. The formula is:

Sum = (n + 1) \* (n / 2) for even 'n'

Sum = (n + 1) \* ((n - 1) / 2) for odd 'n'

Simplifying further, we get:

Sum = n \* (n + 1) / 2

Therefore, the division by 2 in the formula "Sum = n \* (n + 1) / 2" accounts for the fact that each term appears in two pairs when using the pairing technique, and we need to adjust the formula to avoid counting each term twice in the sum. The formula provides the correct sum of the first 'n' natural numbers efficiently and without any doubling effect.

The division by 2 in the formula "Sum = n \* (n + 1) / 2" for the sum of the first 'n' natural numbers arises from a property of arithmetic series and a technique called "pairing" or "averaging."

To understand this, let's consider the arithmetic series of the first 'n' natural numbers:

1 + 2 + 3 + ... + (n-2) + (n-1) + n

If we were to add these numbers one by one, we would get the sum of the series. However, there is a clever way to calculate the sum more efficiently.

Pairing the terms from both ends of the series, we notice the following pattern:

(1 + n) + (2 + (n-1)) + (3 + (n-2)) + ... + ((n-1) + 2) + (n + 1)

Each pair in the parentheses sums to 'n+1', and there are a total of 'n' pairs in the series. So, the sum of the entire series can be expressed as:

Sum = n \* (n + 1)

However, there's a small catch here. When we pair up the terms, we are effectively doubling the sum of the series since each number appears in two different pairs (except the middle term 'n', which remains unpaired in case of an odd value of 'n').

For example, in the series [1, 2, 3, 4, 5], when we pair the terms, we get (1+5), (2+4), and 3 left as it is. The sum is (1+5) + (2+4) + 3 = 3 + 3 + 3 = 9. But the actual sum is 1+2+3+4+5 = 15.

To account for this doubling effect, we divide the result by 2 to get the correct sum of the series:

Correct Sum = (n \* (n + 1)) / 2

By dividing the sum by 2, we effectively "undo" the doubling that occurred when we paired up the terms, giving us the correct sum of the series of the first 'n' natural numbers.

This formula is widely used in various areas of mathematics and computer science to efficiently calculate sums of arithmetic series, including the number of comparisons in algorithms like Bubble Sort.